Course: USO1CPHY02

UNIT -1 ELEMENTARY NERWORK THEORY

1. NETWORK TERMINOLOGY:

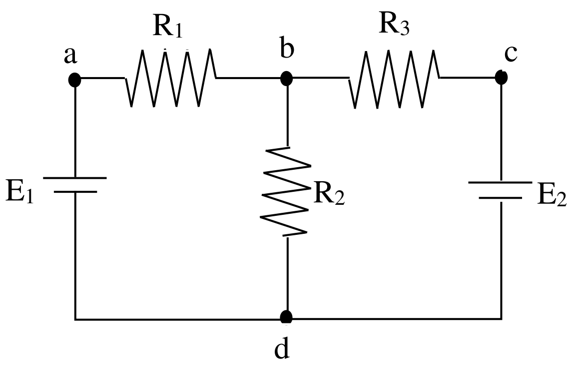


Fig. 1

The following terms are useful in analysis of electrical networks.

1. **Network**: It is an arrangement of passive and/or active elements which form closed paths. In Fig.1 abcda is a network.

2. **Node**: It is a point in a network where two or more circuit elements are joined together. In Fig. 1 a, b, c and d are node points.

3. **Junction**: A junction is that point in a network where three or more circuit elements are joined. In the above network there are two junction points b and d.

4. **Branch**: It is that part of a network which lies between junction points. In the above network bad, bd and bcd are the branches.

5. **Loop**: Any closed path of a network is known as loop. In Fig. 1 abda, bcdb, abcda are loops.

6. **Mesh**: It is the most elementary form of the loop which cannot be further divided into other loops. In Fig. 1 abda and bcda are meshes but abcda is not a mesh because it encloses the first two loops.

7. **Mesh Current**: A mesh current is an imaginary current which flows around a mesh in a clockwise direction. In Fig. 1, I_1 is the mesh current of mesh abda and I_2 is the mesh current of mesh bcdb.

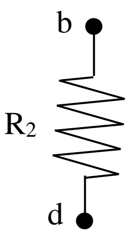


Fig. 2

8. **Tree**: A tree is defined as that part of a network which composed of those branches between the junction points which can be drawn without forming a closed path. In the network of Fig.1 bd forms a tree as shown in Fig 2.

2. VOLTAGE DIVIDER THEOREM:

This theorem is useful to determine voltage drop across a resistor in a series circuit like one as shown in Fig.3.

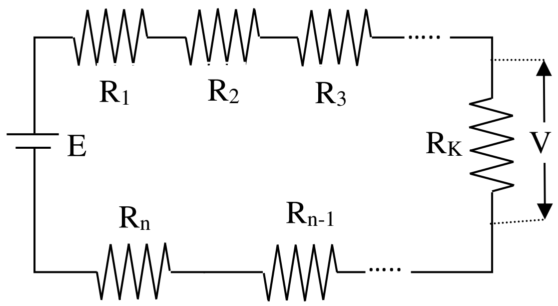


Fig.3

Statement: If n numbers of resistors are connected in series with source of emf E, the voltage drops V_k across the terminals of any resistor R_k of them is given by,

$$V_k = \frac{R_k}{R_1 + R_2 \dots + R_k + \dots + R_n} \text{ x E}$$

This is a voltage divider theorem.

Example 1: Using voltage divider theorem find V in circuit of Fig. 4.

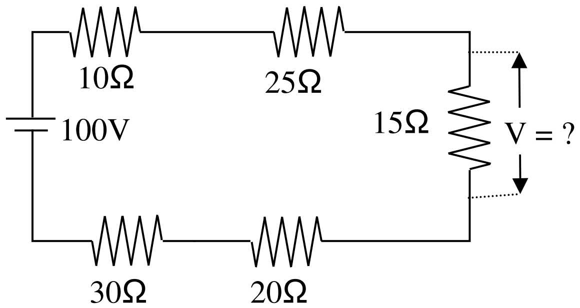


Fig.4

Example 2: Find voltage across resistor of 20Ω in the following circuit of Fig.5.

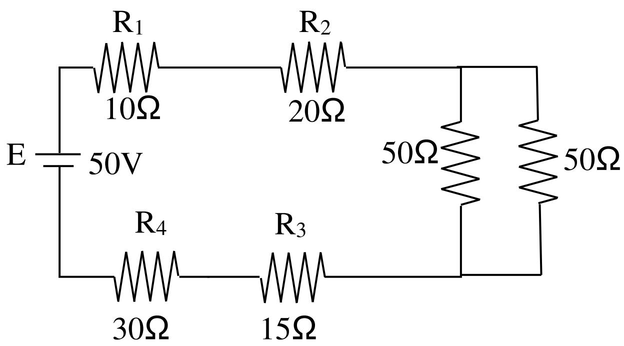


Fig. 5

Solution: The equivalent resistance of 50Ω and 50Ω resistors in parallel is

$$\frac{1}{R_p} = \frac{1}{50} + \frac{1}{50}$$

$$\therefore R_p = \frac{50 \times 50}{50+50} = \frac{2500}{100} = 25\ \Omega$$

This R_p is now in series with other resistors. The voltage across 20 Ω resistor is then given by,

$$V = \frac{R_2}{R_1 + R_2 + R_p + R_3 + R_4} \text{ X E} = \frac{20}{10 + 20 + 25 + 15 + 30} \text{ X } 50$$

$$\therefore V = \frac{20 \text{ X } 50}{100} = 10 \text{ V.}$$

3. SUPERPOSITION THEOREM:

Sometimes we need to find current through or voltage across a particular resistor in a series or linear circuit with two or even more than two sources of emf present in the circuit. The superposition theorem is useful in such cases.

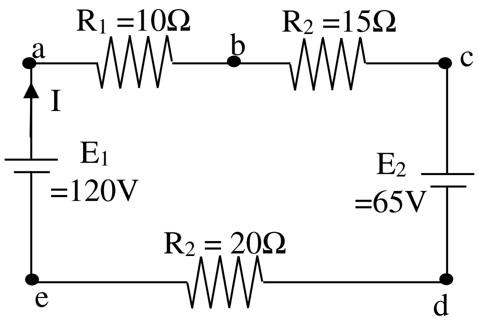


Fig. 6

Explanation:

Consider the circuit shown in Fig.6. It is a series circuit with two batteries (active elements) in series with three resistors (passive elements). Here we assume that loop current I flows in the clockwise direction as shown in the Fig. 6. To find current I in the circuit we apply the Kirchhoff's voltage law to the loop abcdea which gives,

$$IR_1 + IR_2 + IR_3 + E_2 - E_1 = 0$$

$$\therefore \quad I(R_1 + R_2 + R_3) = E_1 - E_2$$

$$\therefore \ I = \frac{E_1 - E_2}{R_1 + R_2 + R_3} \quad \ldots.(i)$$

$$\therefore I = \frac{E_1}{R_1 + R_2 + R_3} - \frac{E_1 - E_2}{R_1 + R_2 + R_3} \quad \ldots.(ii)$$

The first term in equation (ii) i.e., $\frac{E_1}{R_1+R_2+R_3}$ is the current (say I_1) in the circuit due to battery E_1 only i.e. when E_2 is removed and replaced by short circuit between pints c and d. The current will flow in clockwise direction.

The second term in equation (ii) i.e., $\frac{E_2}{R_1+R_2+R_3}$ is the current (say I_2) in the circuit due to battery E_2 only i.e. when E_1 is removed and replaced by short circuit between points a and e. But in this case direction of current will be anticlockwise i.e. opposite to that due to E_1,

When both $\mathbf{E_1}$ and $\mathbf{E_2}$ are present these two currents superimpose in the circuit to give the net or resultant current I. The current I is linearly related to voltage (emf). Following is the statement of superposition theorem.

Statement: If I_1 is the response to E_1 and I_2 is the response to E_2 then, in any linear circuit, the response to the combined function $(E_1 + E_2)$ is $(I_1 + I_2)$.

Example 3: Find the current I flowing in the circuit of Fig.6

For the circuit of Fig.6, Let I_1 be the current due to E_1 only. Then

$$I_1 = \frac{E_1}{R_1+R_2+R_3} = \frac{120}{45} = 2.66 \text{ A}.$$ Here I_1 will flow in clockwise direction.

Similarly, Let I_2 be the current due to E_2 only. Then

$$I_2 = \frac{E_2}{R_1+R_2+R_3} = \frac{65}{45} = 1.44\text{ A}.$$

Here I_2 will flow in anticlockwise direction i.e. opposite to I_1.

According to superposition principle when both $\mathbf{E_1}$ and $\mathbf{E_2}$ are present, the resultant current I will be

$$I = I_1 + I_2 .$$

Since I_2 is opposite to I_1 ,

we get $\quad I = I_1 + (- I_2)$

Hence $I = I_1 - I_2 = 2.66 – 1.44 = 1.22$ A.

Note that I will be in clockwise direction since $\mathbf{E_1} > \mathbf{E_2}$.

Also I can be obtained directly by substituting the given values in equation (i),

$$\therefore I = \frac{120 - 65}{10 + 15 + 20} = \frac{55}{45} = 1.22\text{ A}$$

4. NETWORK ANALYSIS METHODS:

There are two main methods for network analysis namely;
(i)Mesh Current Method and (ii) Node-pair Voltage or Nodal Method.

4.1 Network Analysis by Mesh Current Method

4.1.1 Two-mesh Network:

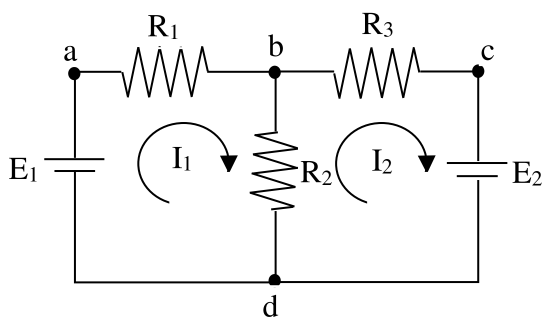


Fig. 7

A network consists of three resistors R_1, R_2, R_3 and two batteries E_1 and E_2 is shown in Fig.7. It consists of two meshes namely abda and bcda. Let I_1 be the mesh current of mesh abda and I_2 be the mesh current of mesh bcdb as shown. Also b and d are the two junction points and bad, bd and bcd are the three branches.

To analyze the circuit using mesh current method, the number of independent mesh equation required m, is given by

$$m = b – (j – 1)$$

where, b = number of branches in the network and
j = number of junction points in the network
In the network of Figure-1 , we have b = 3 and j = 1.

$m = 3 – (2 - 1) = 2.$

Hence two independent mesh equations are required which can be obtained as follow:

For mesh-1 i.e. abda:

Applying Kirchhoff's voltage to the mesh abda we get,

$$I_1R_1 + I_1R_2 - I_2R_2 - E_1 = 0$$

$$\therefore \quad I_1 (R_1+R_2) - I_2R_2 = E_1$$

But $R_1 + R_2 = R_{11}$ = self or total resistance of mesh-1 and

$R_2 = R_{12}$ = mutual resistance of mesh-1 and mesh-2.

Using these, above equation can be written as,

$$\mathbf{I_1 R_{11} - I_2 R_{12} = E_1 \quad ...(1)}$$

For mesh-2 i.e. bcdb:

Applying Kirchhoff's voltage to the mesh bcdb we get,

$$I_2R_3 + E_2 + I_2R_2 - I_1R_2 = 0$$

$$\therefore - I_1R_2 + I_2 (R_2+R_3) = - E_2$$

But $R_2 + R_3 = R_{22}$ = Self or total resistance of mesh-2 and

$R_2 = R_{21}$ = mutual resistance of mesh-2 and mesh-1.

Using these, above equation can be written as,

$$\mathbf{- I_1R_{21} + I_2 R_{22} = - E_2 \quad ...(2)}$$

Equation (1) and (2) are the two requires equations to analyze the circuit using mesh current method. These equations can be solved for I_1 and I_2 if the values of resistances and batteries are known as explain in following example.

Example 4: Find current through R_2 i.e. through branch bd in the circuit of Fig.8

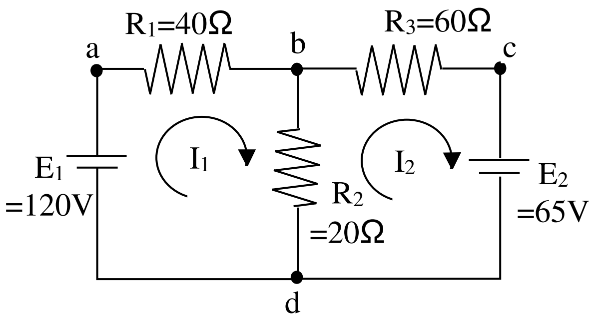


Fig. 8

For mesh-1 i.e. abda:

Applying Kirchhoff's voltage to the mesh abda we get,

$$I_1R_1 + I_1R_2 - I_2R_2 - E_1 = 0$$

$$\therefore \quad I_1 (R_1+R_2) - I_2R_2 = E_1$$

Substituting values of resistances and battery we get

$$I_1 (40 + 20) - I_2 20 = 120$$

$$\therefore \quad 60I_1 – 20I_2 = 120 \quad ….(3)$$

For mesh-2 i.e. bcdb:

Applying Kirchhoff's voltage to the mesh bcdb we get,

$$I_2R_3 + E_2 + I_2R_2 - I_1R_2 = 0$$

Substituting values of resistances and battery we get

$$-20I_1 + 80I_2 = -65 \quad ....(4)$$

Multiplying equation (4) by and 3 and adding to equation (3) we get,

$$-60I_1 + 240I_2 = -195$$
$$60I_1 - 20I_2 = 120$$

$$220\, I_2 = -75$$

$$\therefore I_2 = -\frac{75}{220} = -0.34 \text{ A.}$$

Using this value of I_2 in equation (4) we get,

$$-20I_1 + 80(-0.34) = -65$$
$$-20I_1 + (-27.2) = -65$$
$$-20I_1 = -65 + 27.2 = -37.8$$

$$\therefore I_1 = \frac{37.8}{20} = 1.89 \text{ A.}$$

The current through branch bd i.e. through resistor R_2 is, $I = I_1 + I_2$.
But, since I_2 is opposite to I_1,

$$I = I_1 + (-I_2) = I_1 - I_2$$
$$\therefore I = 1.89 + 0.34 = 2.23\text{A} .$$

This is the through R_2 i.e. through branch bd.

4.1.2 Three-mesh Network:

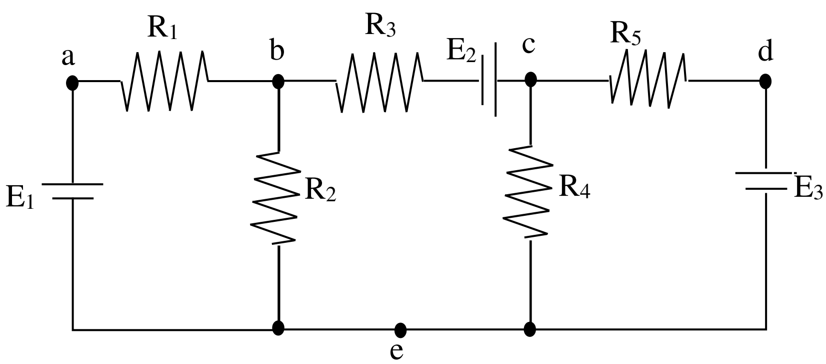


Figure 9.

A network consists of five resistors R_1, R_2, R_3 , R_4 , R_5 and three batteries E_1, E_2 and E_3 is shown in Fig. 9. In the network b, c and e are the three junction points. The network consists of three meshes namely abea, bceb and cdec. Let I_1 be the mesh current of mesh abea, I_2 be the mesh current of mesh bceb and I_2 be the mesh current of mesh cdec as shown. To analyze the circuit using mesh current method, the number of independent mesh equation required m, is given by

$$m = b - (j - 1)$$

where, b = number of branches in the network and j = number of junction points in the network.

In Figure-2 , we have 5 branches namely be, bae, bce, ce and cd. So we have b = 5. Also we have three junction points namely b,c and e. So j = 3.

$\therefore$ m = 5 – (3 - 1) = 3.

Hence three independent mesh equation are required which can be obtained in a similar manner as we followed in case of two- mesh network.

For **mesh-1** i.e. **abea**:

Applying Kirchhoff's voltage we get,

$I_1 (R_1+R_2) - I_2R_2 = E_1$

But $R_1+R_2 = R_{11}$ = self or total resistance of mesh-1 and

$R_2 = R_{12}$ = mutual resistance of mesh-1 and mesh-2.

Using these, above equation can be written as,

$\mathbf{I_1 R_{11} - I_2 R_{12} = E_1}$ **...(i)**

For **mesh-2** i.e. **bceb**:

Applying Kirchoff's voltage we get,

$- I_1R_2 + I_2 (R_2+R_3+ R_4) – I_3R_4 = - E_2$

$\therefore$ $\mathbf{- I_1R_{21} + I_2 R_{22} – I_3R_{23} = - E_2}$ **......(ii)**

where `R_{21} = mutual resistance of mesh-2 and mesh-1.

R_{22} = Self or total resistance of mesh-2.

R_{23} = mutual resistance of mesh-2 and mesh-3.

For **mesh-3** i.e. **cdec**: Applying Kirchoff's voltage we get,

$\therefore$ $- I_2R_3 + I_3 (R_4+ R_5) = - E_3$

$\therefore$ $\mathbf{- I_2R_{32} + I_3 R_{33} = - E_3}$ **......(iii)**

where R_{32} = mutual resistance of mesh-3 and mesh-3.

R_{33} = Self or total resistance of mesh-3.

The equation (i) ,(ii) and (iii) can be solved to determine I_1,I_2 and I_3.

4. 2 Network Analysis by Node-pair Voltage or Nodal Method.

4.2.1 One-node pair network.

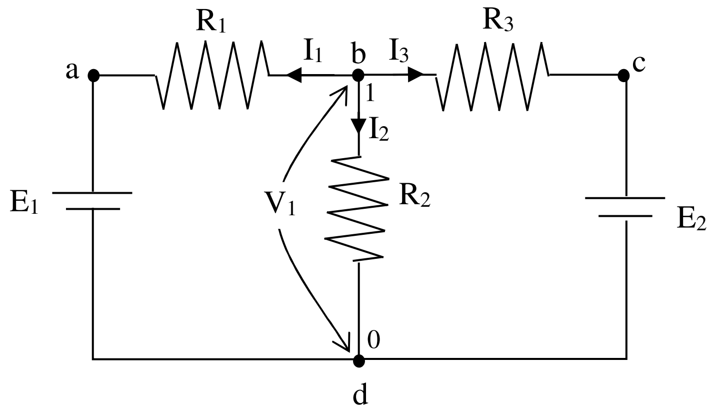


Fig. 10

A network consists of three resistors R_1, R_2, R_3 and two batteries E_1 and E_2 is shown in Fig. 10. As shown b and d are the two junction points as well as node points labeled as point 1 and 0 respectively. The pair of points 1 and 0 forms a single node pair in the network.

Let the node potential between these points be V_1 as shown in the figure. Also assume that V_1 is the highest potential in the network so that branch currents I_1 , I_2 and I_3 are leaving the points 1 as shown.To analyze the circuit using node voltage method, the number of independent node equation required n, is given by,

n = j – 1.

where, j = number of junction points in the network.

Since we have j = 2, we get n = 2 - 1 = 1.
Hence one nodal equation is required which can be obtained as follows.
Applying Kirchoff's Current law to the node point 1 we get,

$\mathbf{I_1 + I_2 + I_3 = 0}$ **....(i)**

Applying KVL to branch bad we get, $V_1 = I_1R_1 + E_1$

$$\therefore\ I_1 = \frac{V_1 - E_1}{R_1} \qquad \text{... (ii)}$$

Applying KVL to branch bd we get, $V_1 = I_2R_2$

$$\therefore\ I_2 = \frac{V_1}{R_2} \qquad \text{... (iii)}$$

Applying KVL to branch bcd we get, $V_1 = I_3R_3 + E_2$

$$\therefore\ I_3 = \frac{V_1 - E_2}{R_3} \qquad \text{... (iv)}$$

Using equation (ii), (iii) and (iv) in (i) we get

$$\frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - E_{21}}{R_3} = 0$$

$$\therefore\ V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{E_1}{R_1} + \frac{E_2}{R_3} \qquad \text{... (v)}$$

Here $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$ is the self conductance of node 1 = G_{11} .

$\therefore\ V_1\,G_{11} = \frac{E_1}{R_1} + \frac{E_2}{R_3}$ This is the required equation.

Example 5: In the network of Fig.11 find I_2 through R_2.

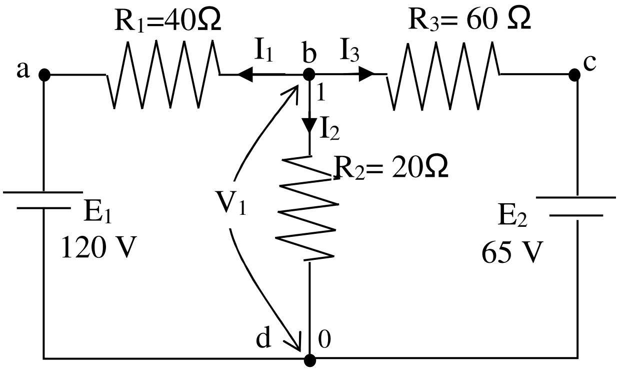


Fig.11

Using equation (v), we get

$$V_1\left(\frac{1}{40} + \frac{1}{20} + \frac{1}{60}\right) = \frac{120}{40} + \frac{65}{60}$$

$$\therefore\ V_1\,(\,0.091) = 4.08$$

$$\therefore\ V_1 = 44.83\ \text{V}$$

Since $I_2 = \frac{V_1}{R_2}$, we get $I_2 = \frac{44.83}{20} = 2.24$ A.

4.2.2 Two-node pair network.

A network consists of three resistors R_1, R_2, R_3, R_4, R_5 and three batteries E_1 E_2 and E_3 is shown in Fig. 12. As shown b, c and e are the three junction points as well as node points. The pair of points 1(or b) and 0(or e) forms a node-1in the network and the pair of points 2(or c) and 0(or e) forms a node-2 in the network. Let the potential of node-1 be V_1 and that of node-2 be V_2 as shown in the Fig. 12.

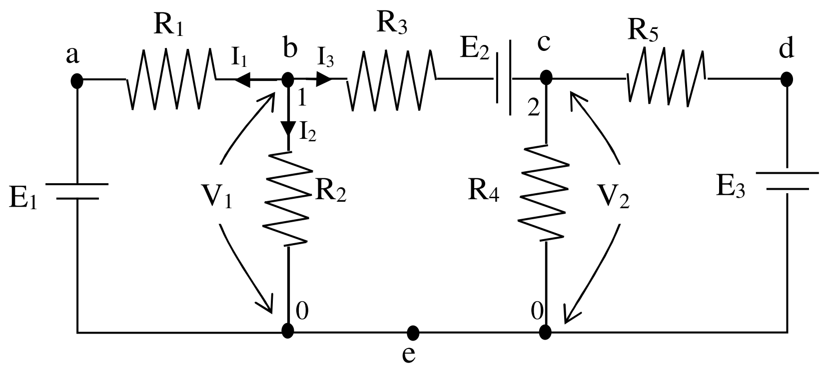


Fig. 12

To analyze the circuit using node voltage method, the number of independent node equation required n, is given by, $n = (j - 1)$ where, j = number of junction points in the network.

Since we have $j = 3$, we get $n = (3 - 1) = 2$.

Hence, two nodal equations are required which can be obtained as follows.

For node-1:

We assume that V_1 is the highest potential in the network so that branch currents I_1 , I_2 and I_3 are leaving the point 1 as shown in Figure-4. Applying Kirchhoff's Current law to the node point 1 we get,

$$\mathbf{I_1 + I_2 + I_3 = 0} \quad \text{....(i)}$$

Applying KVL to branch bae we get, $V_1 = I_1R_1 + E_1$

$$\therefore\ I_1 = \frac{V_1 - E_1}{R_1} \qquad \text{... (ii)}$$

Applying KVL to branch be we get, $V_1 = I_2R_2$

$$\therefore\ I_2 = \frac{V_1}{R_2} \qquad \text{... (iii)}$$

Applying KVL to branch cde we get, $V_1 = I_3R_3 - E_2 + V_2$

$$\therefore\ I_3 = \frac{V_1 - V_2 + E_2}{R_3} \qquad \text{... (iv)}$$

Using equation (ii), (iii) and (iv) in (i) we get

$$\frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2 + E_2}{R_3} = 0\,.$$

$$\therefore\ V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - V_2\left(\frac{1}{R_3}\right) = \frac{E_1}{R_1} - \frac{E_2}{R_3} \qquad \text{... (v)}$$

Here $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$ is the self conductance of node 1 = G_{11} .

and $\left(\frac{1}{R_3}\right)$ is the mutual conductance of node 1 & 2 = G_{12} .

$$\therefore\ V_1\, G_{11} - V_2\, G_{12} = \frac{E_1}{R_1} + \frac{E_2}{R_3} \qquad \text{... (a)}$$

This is the first required equation.

For node-2:

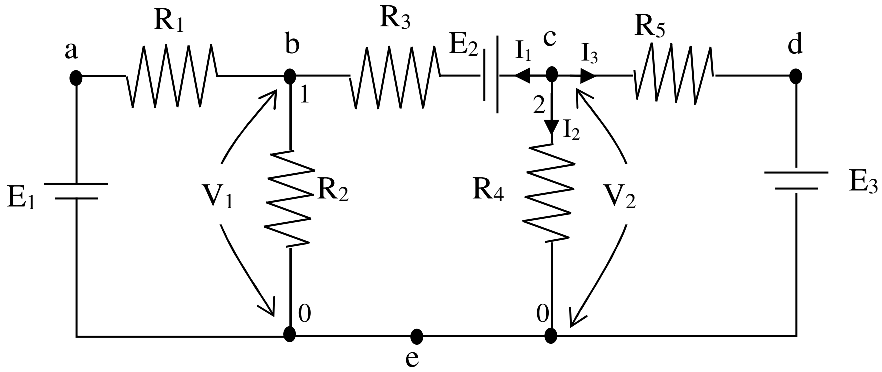


Fig. 13

In this case we assume that V_2 is the highest potential in the network so that current I_1 through branch cb, I_2 through branch ce and I_3 through branch cde are leaving the point 2(or c) as shown in Fig. 13.

Applying Kirchhoff's Current law to the node point c we get,

$$\mathbf{I_1 + I_2 + I_3 = 0} \qquad \text{....(i)}$$

Applying KVL to branch cb we get, $V_2 = E_2 + I_1R_3 + V_1$

$$\therefore\ I_1 = \frac{V_2 - V_1 - E_2}{R_3} \qquad \text{... (ii)}$$

Applying KVL to branch ce we get, $V_2 = I_2R_4$

$$\therefore\ I_2 = \frac{V_2}{R_4} \qquad \text{... (iii)}$$

Applying KVL to branch cde we get, $V_2 = I_3R_5 + E_3$

$$\therefore\ I_3 = \frac{V_2 - E_3}{R_5} \qquad \text{... (iv)}$$

Using equation (ii),(iii) and (iv) in (i) we get,

$$\frac{V_2 - V_1 - E_2}{R_3} + \frac{V_2}{R_4} + \frac{V_2 - E_3}{R_5} = 0$$

$$\therefore\ -V_1\left(\frac{1}{R_3}\right) + V_2\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) = \frac{E_2}{R_3} + \frac{E_3}{R_5} \qquad \text{..(v)}$$

Here $\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)$ is the self conductance of node 2 = G_{22} .

and $\left(\frac{1}{R_3}\right)$ is the mutual conductance of node 2 & 1 = G_{21} .

$$\therefore\ -V_1\,G_{21} + V_2\,G_{22} = \frac{E_2}{R_3}+\frac{E_3}{R_5} \quad \text{... (b)}$$

The equation (a) and (b) are the two required equations.

5. Thevenin's Theorem

This theorem is useful to:

(i) convert a complicated network into a simple series circuit and

(ii) find branch current when branch resistance changes frequently.

Statement: Any two terminals of a network composed of active and passive circuit elements can be replaced by:

(i) an equivalent voltage source, which is the voltage across that two terminals with all external elements disconnected and

(ii) an equivalent series resistance, which is the equivalent resistance between that two terminals with all sources of emf shorted.

Consider network of Fig.14. As shown E_b is the emf, of battery, r_i is the internal resistance of the battery, S is a switch and R_L is the load resistance.

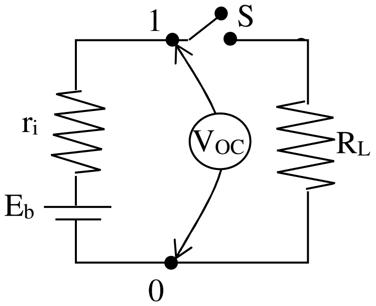


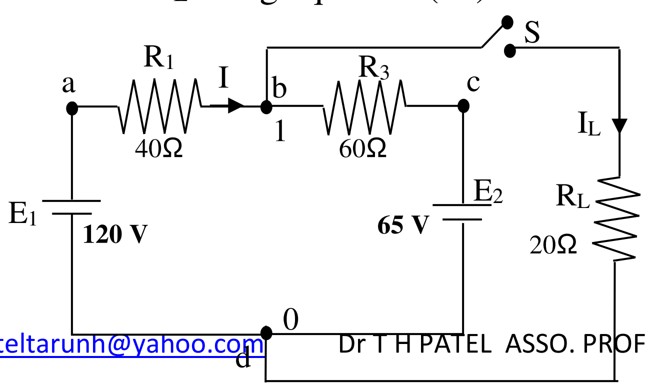
Fig. 14

When S is open, the voltage across terminal 1 and 0 is the open circuit voltage V_{OC} which will be equal to the emf of battery E_b. i.e. $V_{OC} = E_b$.

When S is closed current I_L flows through r_i and R_L. Applying Kirchoff's voltage law we get

$$E_b = I_L r_i + I_L R_L \qquad \therefore\ I_L = \frac{E_b}{(r_i+R_L)} \quad \text{...(25)}$$

Hence, once E_b and r_i are known, I_L can be determined easily for different values of R_L using equation (25).



Application/Example 6:

Suppose we want to find the I_L for different values of R_L in the network of Fig. 15. Then using Theninenin's theorem we can convert this complicated network into simple series circuit in following way.

Fig. 15

Step-1. Determine open circuit voltage V_{01} across terminals 1 and 0 (i.e. with R_L disconnected.)

For this let S be open in Figure 7. Now let I be the current through loop abcda. Then using superposition theorem we get,

$$I = \frac{E_1 - E_2}{R_1 + R_3} \quad \text{... (i).}$$

Putting values for network of Figure 7,

$$I = \frac{120 - 65}{40 + 60} = \frac{55}{100} = 0.55 \text{ A.}$$

The open circuit voltage V_{01} is given as, $V_{01} = E_1 - IR_1 = E_2 + IR_3$.

$\therefore$ $V_{01} = 120 - (0.55)(40) = 120 – 22 = 98$ V.

This the value of open circuit voltage across terminal 1 and 0.

Step 2. Determine equivalent series resistance R_i between terminal 1 and 0 with all sources of emf shorted.

For this let's short terminal a with d and c with d. As a result of this R_2 and R_3 becomes parallel to each other as shown in Figure 8. The equivalent resistance $\mathbf{R_i}$ between 1 and 0 is then given by

$$r_i = \frac{R_1 R_3}{R_1 + R_3} \quad \text{.... (ii)}$$

Putting values for network of Figure 7, we get

$$r_i = \frac{40 \text{ X } 60}{40 + 60} = \frac{2400}{100} = 24\ \Omega$$

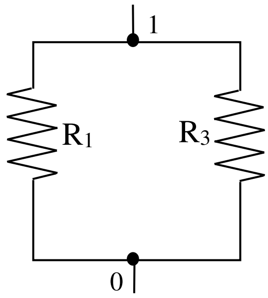
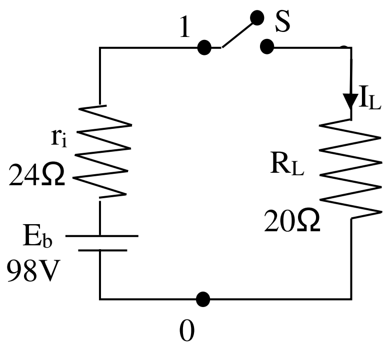


Fig.16

Step-3. Construction of final series circuit.

Using Thevenin's theorem, the circuit across terminals 1 and 0 of Fig.15 can be replaced as shown in Fig. 17, which is a simple series circuit.



The current I_L is then ,

$$I_L = \frac{E_b}{(r_i + R_L)}$$

$$\therefore \quad I_L = \frac{98}{(24 + 20)} = 2.22 \text{ A .}$$

Fig.17

Example 7: Find I_L when R_L = 40Ω, 55Ω, and 60Ω.

6. Norton's Theorem:

This theorem explains about the current equivalent of a voltage source.It was first developed by E.L. Norton at the Bell Telephone laboratory. This theorem is an alternative to the Thevenin's theorem. Whereas Thevenin's theorem reduces a two terminal active network of liner resistance and sources to an equivalent constant voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant current source and a parallel resistance. Norton's theorem is very much useful for the circuit containing transistor and other devices which are current sensitive.

Statement:

"Any two terminals of a network composed of liner passive and active circuit elements may be replaced by (i) an equivalent current source and (ii) a parallel resistance. The current of the source is the current measured in the short circuit placed across the terminal pair. The parallel resistance is the equivalent resistance looking into the terminal pair with all independent power sources inactive."

Explanation:

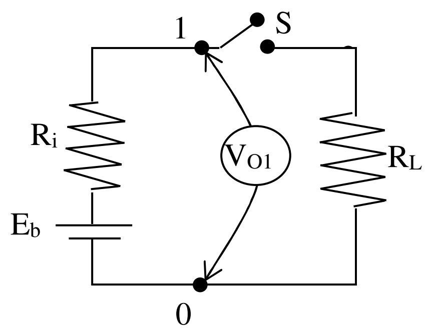


Fig.18

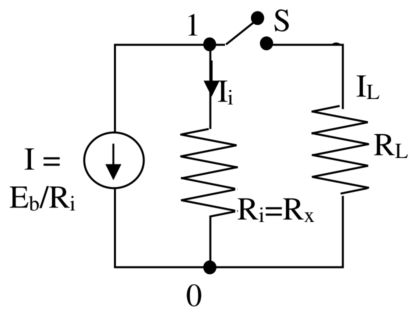


Fig. 19

Consider the circuit of Fig.18. When R_L is zero, a current flows through the output terminals from 1 to 0. This current is maximum possible current that can be supplied by battery or given voltage source. Hence we can consider the voltage source as a source of current.

Let I be the maximum current through R_i then we can write, $I = \frac{E_b}{R_i}$ ....(i).

Now this current source must provide the same voltage across terminal 1 and 0 of Fig.18 i.e. V_{01}. This can be done by connecting a shunt resistance R_x across the terminal 1 and 0 as shown in Fig. 19. Here Fig.19 shows the current source equivalent of a voltage source.

Now when S is open we get from Fig. 18, $IR_x = V_{01}$

Also from Fig.1. $V_{01} = E_b = IR_i$.

So we can write, $IR_x = V_{01} = E_b = IR_i$ $\therefore \mathbf{R_x = R_i}$.

Hence as long as S is open, the left part of terminal 1 and 0 of Fig. 18 can be replaced by an equivalent current source of magnitude $I = E_b/R_i$ and a shunt resistance of value R_i.

Now applying KVL to Fig.18, when S is closed, We get,

$$E_b = I_L R_i + I_L R_L$$

$$\therefore E_b - I_L R_i = I_L R_L$$

$$\therefore \frac{E_b}{R_i} - I_L = \frac{V_{01}}{R_i}$$

$$\therefore E_b - I_L R_i = V_{01}. \,(V_{01} = I_L R_L)$$

Since $E_b/R_i = I$, we get, $I - I_L = \frac{V_{01}}{R_i}$ (ii)

Using this expression, I_L can be determinate.

Applying KCL to node point 1 of Fig. 19 we get,

$$I = I_i + I_L \qquad \therefore I = \frac{V_{01}}{R_i} + \frac{V_{01}}{R_L}$$

$$\therefore I = V_{01}\left(\frac{1}{R_i} + \frac{1}{R_L}\right) = \frac{V_{01}}{R_p}$$

$$\therefore V_{01} = I \times R_p \qquad \text{... (iii)}$$

Here I is equivalent current and R_p is equivalent resistance. Using above expression terminal voltage can be determined.

Example 8:

Find current through R_L for R_L =11, 20 and 30Ω in circuit of Fig. 20.

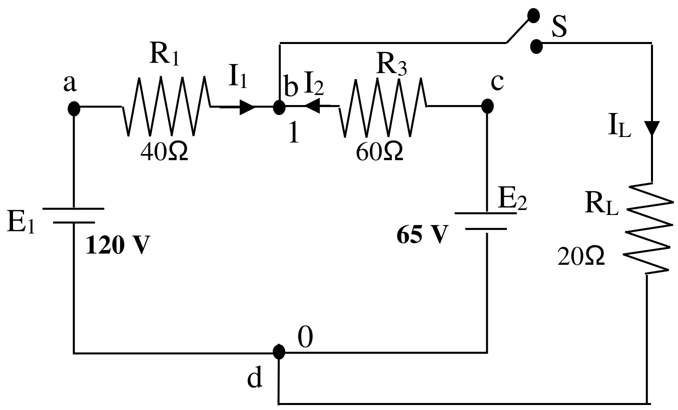


Fig. 20

From the diagram we have,

$I_1 = \frac{E_1}{R_1} = \frac{120}{40} = 3.0\text{ A}$ & $I_2 = \frac{E_2}{R_3} = \frac{60}{65} = 1.08\text{ A}$.

The total current through the short-circuit across terminals 1 & 0 is then,

$I = I_1 + I_2 = 3.00 + 1.08 = 4.08\text{ A}$ …(i)

Now, equivalent resistance across terminals 1 & 0 is,

$R_i = R_1 \parallel R_3$

$$\therefore R_i = \frac{R_1 R_3}{R_1 + R_3} = \frac{40 \times 60}{40 + 60} = \frac{2400}{100} = 24\,\Omega \quad \text{... (ii)}$$

Hence, using values in equation (i) & (ii) we get equivalent circuit of Fig. 20 as shown in Fig. 21.

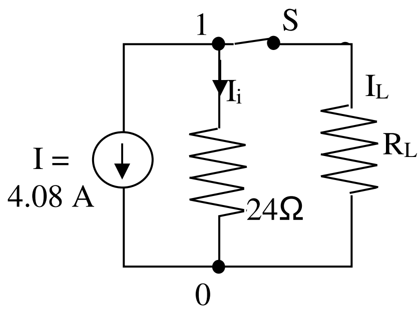
Applying KCL to point 1 in Fig. 21 when S is closed, we get,

$I = I_i + I_2$

$\therefore I_i = I - I_2$

Also, $I_i R_i = I_2 R_2$

$\therefore I_i R_i = (I - I_2) R_i$



Using these results, we can write,

$I_2 R_2 = (I - I_2) R_i = I R_i - I_2 R_i$

$\therefore I_2 R_2 + I_2 R_i = I R_i$

$\therefore I_2 (R_2 + R_i) = I R_i$

$\therefore I_2 = \frac{R_i}{R_2 + R_i} I$ ...(iii)

Using above relation we get.

$I_2 = \frac{24}{24+11} \times 4.08 = 2.8\text{ A}$

Similarly we can find I_2 for $R_2 = 20$ & $30\,\Omega$.

ASSIGNMENT UNIT- 1 US01CPHY02

A Multiple Choice Questions:

1	A point in a network where three or more circuit elements are joined together is call as ….. **(a) Junction** (b) node (c) branch (d) tree.
2	Which current always flows in a clockwise direction, **(a) mesh current** (b) loop current (c) branch current (d) node current.
3	How many number of independent node equations are required to analyze a network

	having three junction point and five branches? (a) 2 (b) 6 (c) 5 **(d) 3**
4	How many number of independent mesh equations are required to analyze a network having three junction point and five branches? (a) 2 (b) 6 **(c) 5** (d) 3
5	When two resistors are connected in parallel, their equivalent resistance .... (a) increases (b) may not change **(c) decreases** (o) becomes zero.

B. Short answer questions.

1	State and explain voltage divider theorem.
2	What is superposition principle ? Give its statement.
3	Three resistors, R_1=10 Ω, R_2=20 Ω and R_3=30 Ω are connected in series with a batter of 10V. Find the voltage drop across R_2.
4	Three resistors, R_1=10 Ω, R_2=20 Ω and R_3=30 Ω are connected in parallel with a batter of 10V. Find the current passing through R_2.
5	State Thevenin theorem.
6	State Norton theorem.

C. Long answer questions.

1	With proper example explain mesh current analysis of two mesh network .
2	Discuss nodal method of network analysis for a two node pair network.
3	Explain: Thevenin's theorem.
4	Using necessary network diagram explain network terminology.
5	With proper example explain nodal analysis of one node pair network .
6	With proper network explain Norton theorem.
7	Give a detailed comparison of Thevenin and Norton theorem and state the benefits of Norton's theorem.

EXCERSICE
MULTIPLE CHOICE QUESTIONS

1. Superposition theorem is useful in analyzing a network containing
   (a) one voltage source (b) one resistor
   (c) more than one voltage sources (d) more than one resistors
2. Superposition theorem is applicable to the networks in which the current and voltage has

(a) linear relation (b) exponential relation

(c) logarithmic relation (d) quadratic relation

3. For two resistors connected in series with each other, the following quantity will be the same

   (a) voltage **(b) current** (c) frequency (d) amplitude

4. For two resistors connected in parallel with each other, the following quantity will be the same

   (a) voltage (b) current (c) frequency (d) amplitude

5. Every junction is a _______ . Complete the sentence choosing the proper word.

   (a) node (b) mesh (c) loop (d) branch

6. Every node may not be a _______ . Complete the sentence choosing the proper word.

   (a) mesh **(b) junction** (c) loop (d) branch

7. Every mesh is a _______ . Complete the sentence choosing the proper word.

   (a) node (b) mesh **(c) loop** (d) branch

8. Every loop may not be a _______ . Complete the sentence choosing the proper word.

   (a) mesh (b) junction (c) loop (d) branch

9. Node is a point of a network where the number of components connected are

   (a) one (b) two **(c) two or more** (d) one or more

10. Junction is a point of a network where the number of components connected are

    (a) three (b) two (c) two or more **(d) three or more**

11. Mesh method of network analysis makes use of the following to analyse a network.

    (a) Kirchoff's Voltage Law (b) Kirchoff's Current Law

    (c) Faraday's Law (d) Lenz's Law

12. Node-Pair voltage method of network analysis makes use of the following to analyse a network.

    (a) Kirchoff's Voltage Law **(b) Kirchoff's Current Law**

    (c) Faraday's Law (d) Lenz's Law

13. The reciprocal of resistance is called

    (a) impedance (b) reactance **(c) conductance** (d) reluctance

14. In Thevenin's theorem V_{oc} refers to

    (a) open component voltage
    (b) open controlled voltage
    (c) open circuit voltage

(d) on component voltage

15. In Thevenin's theorem I_{sc} refers to

(a) short circuit current

(b) short component current

(c) single circuit current

(d) single component current